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Measurement of strain and axial contraction of an FRP tube under large rotation

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Abstract—Large strain measurement and axial contraction of a fiber-reinforced plastic (FRP) tube under biaxial loading were investigated theoretically and experimentally. The equation for calculating strains from strain gage measurements for the case of large deformation and large rotation was derived based on the analysis of kinematics of deformation. Axial contraction of a tube could be predicted as a function of stresses, a twist angle and an initial elastic modulus of the axial direction. To demonstrate applicability of the equations, the tests of a cross-ply carbon/epoxy composite tube were conducted under uniaxial and tension/torsion biaxial loading up to failure. The signal of a linear variable differential transformer (LVDT) and angular displacement transducer (ADT) of a testing machine could be adopted in strain measurement of the biaxial test for fairly large deformation. The prediction of axial contraction of a CFRP tube agreed well with the LVDT result of the testing machine.

Keywords: Carbon/epoxy composite (CFRP); biaxial loading; large deformation; strain gage measurement; kinematics of deformation; axial contraction of CFRP tube.

1. INTRODUCTION

The wire or foil type strain gage and extensometer are useful instruments to measure strains. Among them, an electrical-resistance metallic foil strain gage is the most widely used in the measurement of dynamic deformation as well as static deformation due to its high resolution and a strong installation. The foil gage produced from a parent metal foil is normally bonded to a thin polymeric backing material, and the entire assembly is adhesively bonded to the test structure. The accuracy and range of measurement are increased due to improvements in gage resistance and sensitivity tolerances, a decrease in gage thickness, and because of the wide variety of grid shapes that can easily be produced using the photoetch process. As a result, the maximum strain which can be obtained in experiments has been increased from

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around several percent up to 20–30 percent recently. Therefore, it is important to establish the validity of the use of electrical-resistance foil strain gages for measuring large strain. Huang *et al.* [1] showed that the gage factor of the strain gage varies linearly with strain and measurement of large strain with electrical-resistance foil strain gages is valid. Thus the measurement of large strain when the gage axis coincides with respect to the intended direction of strain measurement may be possible in most experiments. When the shear strain to be measured is large, however, a problem still remains. The strain gage mounted on the surface of the specimen will rotate with it by a large rotation and the angles between the gage and the basic axis direction will deviate from their initial values. The strain equation using a rosette gage, established based on the assumption of small deformation, may be uncertain for large deformation. The authors have derived Lagrangian finite strain relations based on the analysis of kinematics of deformation for the case of large deformation and large rotation [2].

Although the strain of isotropic materials can be measured with such strain gages and an extensometer having a certain gage length, unnecessary movement of the extensometer and unusual behavior of the strain gage [3], which would not be expected based upon experience with isotropic materials, prevents us from measuring the strain of fiber reinforced plastic composite materials (FRP). Several studies [4–7] have reported on the measurement of resultant strain of composite materials. In [4], the points of maximum and minimum deformation were measured by reading the return points of the ball screw rotation of an Instron type of machine (Shimazu Autograph IS-10T). Then the axial strain was calculated from the results. In [5] and [6], residual elongations of unidirectional composites were measured from the output of a linear variable differential transformer (LVDT) of the MTS machine. This deformation was then converted to strain within the gage length by using a calibration curve established separately. Huang *et al.* [7] also established that the strain measurement using the output of an angular displacement transducer (ADT) of the MTS machine can be done reasonably up to a shear strain of 30% compared with those of strain gage technique for extruded pure copper.

This paper intends to investigate the validity of the assumption of infinitesimal deformation and the measurement error of calculation directly from the LVDT or ADT signal of a testing machine, for measuring large deformation in an FRP structure. Axial contraction of an FRP under pure torsion and biaxial loading is considered also. In general, axial stretch is different from unity for the tension-plane torsion experiment. Even in the case of plane torsion load, it is well-known that pure shear is accompanied by axial extension or contraction, as discussed by Neale *et al.* [8]. In addition, axial strains in a braided rod loaded in torsion could be both tensile and compressive, depending on the sign of torque or weaving step [9].

In this study, a new equation for strain measurement for the case of large deformation and large rotation was derived based on the analysis of kinematics of deformation. Also, axial contraction of an FRP tube under pure torsion and biaxial loading was investigated theoretically and experimentally.

2. THEORETICAL ANALYSIS

2.1. Measurement of large strains

When the shear strain to be measured is large, the strain gage mounted on the surface of a specimen will rotate with it and the angles between the gage and the x-axis direction will deviate from their initial values. An equation for strain for the case of large deformation and large rotation was developed, in Appendix based on the analysis of kinematics of deformation:

$$(1+\varepsilon_\theta)^2 = \alpha^2 \cos^2\theta + \left(\beta^2+\gamma^2\right)\sin^2\theta + 2\alpha\gamma \sin\theta\cos\theta, \tag{1}$$

where θ is the inclined angle of strain gages to the x-axis in the undeformed configuration as shown in Fig. 1. It shows deformation of a tube under multi-axial loadings. ε_θ is the strain measured by a strain gage inclined at θ degree to the x-axis in the undeformed configuration. α, β and γ are the elements of the deformation gradient tensor, which can be given as a function of measured strains.

In general, the infinitesimal strains (e_x, e_y and e_{xy}) can be calculated by the following equations, which can be derived from equation (A11), under the assumption of infinitesimal deformation and negligible change of the gage orientations:

$$e_x = \varepsilon_1 + \varepsilon_3 - \varepsilon_2, \tag{2a}$$

$$e_y = \varepsilon_2, \tag{2b}$$

$$2e_{xy} = \varepsilon_1 - \varepsilon_3, \tag{2c}$$

where ε_1, ε_2 and ε_3 are the strains measured by strain gages inclined at 45°, 90° and 135° to the x-axis, respectively. Substituting strains measured by strain gages

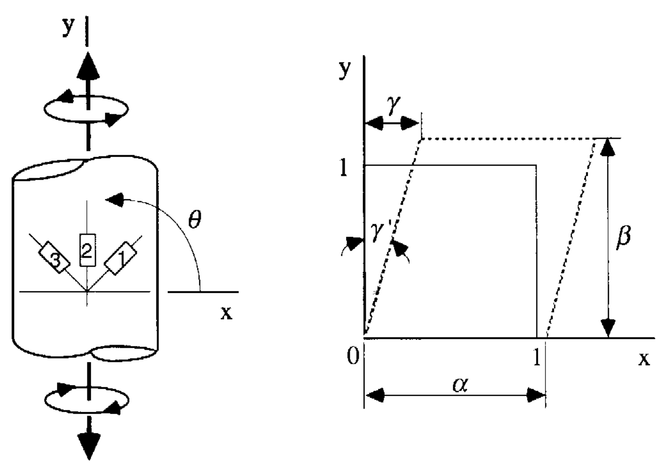

Figure 1. Strain gage location and deformation under multi-axial loading.

arranged as shown in Fig. 1, $\theta_1 = 45°$, $\theta_2 = 90°$ and $\theta_3 = 135°$, and its angle into equation (1), respectively, leads to the following three relations among the nominal strains (ε_1, ε_2 and ε_3) and deformation gradient tensor (α, β and γ):

$$\begin{aligned}
\alpha &= \left[1 + 2(\varepsilon_1 + \varepsilon_3 - \varepsilon_2) + \left(\varepsilon_1^2 + \varepsilon_3^2 - \varepsilon_2^2\right)\right]^{0.5} \\
\beta &= \left[(1 + \varepsilon_2)^2 - \gamma^2\right]^{0.5} \\
\gamma &= (1/\alpha)\left[(\varepsilon_1 - \varepsilon_3) + 0.5\left(\varepsilon_1^2 - \varepsilon_3^2\right)\right].
\end{aligned} \tag{3}$$

Lagrangian finite strain, $\boldsymbol{E}$ is defined [10] and can be expressed using equation (A8) in terms of α, β and γ, as follows [2]:

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^T \cdot \boldsymbol{F} - \boldsymbol{I}) = \frac{1}{2}\begin{bmatrix} \alpha^2 - 1 & \alpha\gamma \\ \alpha\gamma & \beta^2 + \gamma^2 - 1 \end{bmatrix} = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_y \end{bmatrix}, \tag{4}$$

where, $\boldsymbol{I}$ is unit tensor and ε_x, ε_y and ε_{xy} are the Lagrangian components of the finite strain tensor. Substituting equation (3) into equation (4), one obtains

$$\varepsilon_x = (\varepsilon_1 + \varepsilon_3 - \varepsilon_2) + 0.5\left(\varepsilon_1^2 + \varepsilon_3^2 - \varepsilon_2^2\right), \tag{5a}$$

$$\varepsilon_y = \varepsilon_2 + 0.5\varepsilon_2^2, \tag{5b}$$

$$2\varepsilon_{xy} = (\varepsilon_1 - \varepsilon_3) + 0.5\left(\varepsilon_1^2 - \varepsilon_3^2\right). \tag{5c}$$

In the case of plane loading, the e_x and e_y can be interpreted as extensions of vectors originally parallel to the coordinate axes, while e_{xy} represents shear or change of the angle between vectors originally at right angles. When the strain components are large, however, it is no longer possible to give simple geometrical interpretations of the function ε_{ij} [11]. In fact, it is the most useful to obtain the strains using the displacement from the output of the LVDT or ADT of the MTS machine. The axial and shear strain can be then calculated by the following equations:

$$\varepsilon_y = \Delta L/L, \tag{6a}$$

$$2\varepsilon_{xy} = \phi R/L, \tag{6b}$$

where L is the current (extended) gage length, ΔL is the difference between the extended gage length and original gage length, R is the mean radius of the specimen and ϕ is the relative rotation angle of two ends of the gage length section.

2.2. *Axial contraction of a tube by rotation*

Two dimensional rectangular block OABC subjected to biaxial loading is shown in Fig. 2a, schematically. Suppose that fiber $\overline{\mathrm{MN}}$ at undeformed state becomes $\overline{\mathrm{MN}'}$ at a deformed state. Single fiber deformation is presented in Fig. 2b. The relations between undeformed and deformed states can be expressed as follows:

$$\begin{aligned} L_\mathrm{f} &= \overline{\mathrm{MN}} = \overline{\mathrm{MP}} \\ L_\mathrm{f}' &= \overline{\mathrm{MQ}} = \overline{\mathrm{MP}} + \Delta L_\mathrm{f}' = L_\mathrm{f}/\cos\gamma' \\ \Delta L_\mathrm{f}' &= \Delta L_\mathrm{f} + C_\mathrm{f}, \end{aligned} \tag{7}$$

where γ' is a twist angle and C_f is the contraction of fiber by fiber stiffness. It is assumed that the extension of fiber at a deformed state occurs only by load parallel

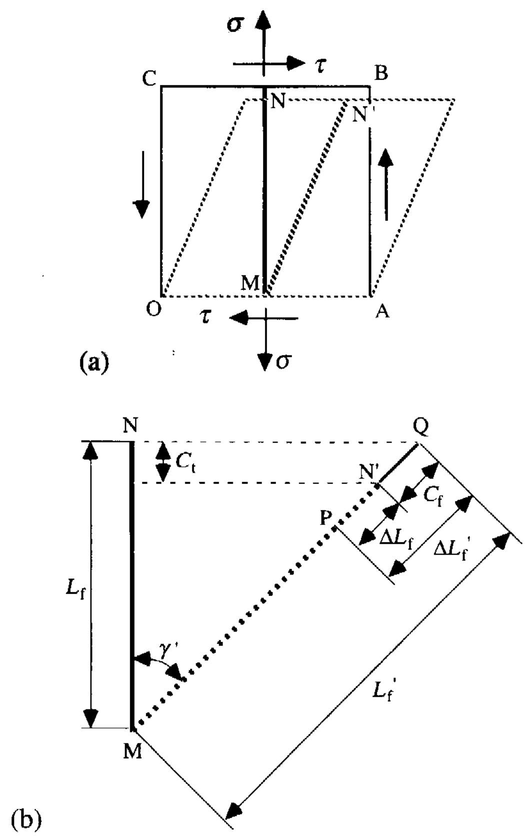

Figure 2. Deformation under biaxial loading: (a) unit area element deformation; (b) single fiber deformation.

to the fiber direction. Thus the strain of fiber under biaxial loading can be written as

$$\varepsilon^* = \Delta L_{\mathrm{f}}/L_{\mathrm{f}} = \varepsilon_0 \sigma^*/\sigma_0, \tag{8}$$

where σ_0 and ε_0 are the axial stress and strain under the linear relation at pure tension, respectively. σ^* is the load parallel to the fiber direction, which can be calculated by stress transformation:

$$\sigma^* = \sigma \cos^2 \gamma' + \sigma' \sin^2 \gamma' + 2\tau \sin \gamma' \cos \gamma', \tag{9}$$

where σ' is circumferencial stress. Equations (7) and (8) give the deformation of fibers as

$$\begin{aligned} C_{\mathrm{f}} &= \Delta L'_{\mathrm{f}} - \Delta L_{\mathrm{f}} \\ &= \left(L_{\mathrm{f}}/\cos\gamma' - L_{\mathrm{f}}\right) - L_{\mathrm{f}}\sigma^*\varepsilon_0/\sigma_0. \end{aligned} \tag{10}$$

Axial contraction of tube can be obtained as

$$C_{\mathrm{t}} = C_{\mathrm{f}} \cos \gamma'. \tag{11}$$

Applying Hooke's law in axial loading through equations (9) to (11) and letting the original length unity ($L_{\mathrm{f}} = 1$), the axial contraction of tube under tension/torsion biaxial loading can be expressed as follows:

$$C_{\mathrm{t}} = 1 - \cos \gamma' - E^{-1}\left(\sigma \cos^3 \gamma' + 2\tau \sin \gamma' \cos^2 \gamma'\right), \tag{12}$$

where E is modulus of elasticity in the axial direction. For simplicity, it is considered to be constant.

3. EXPERIMENTAL

Uniaxial and biaxial tests were performed with MTS 809 Axial-Torsional Test Systems. The specimen was a cross-ply composite tube made by the lapped moulding technique. The fiber was T300 grade carbon fiber and epoxy resin was used as matrix. The prepregs with 0° and 90° layers were laminated with an overlap. After curing, the tubes were cut into 100 mm length and glass cloth end-tabs with 35 mm long and 30 mm outside diameter were attached to the outside of the tube. The specimens were grained on a lathe using abrasive paper to remove marks left by shrinkage of film and to adjust the tube thickness. Aluminum liners were inserted into the inside of specimen ends. The grips and the end reinforcement were designed to minimize stress concentrations in the test section. The specimens have a inner diameter of 20 mm, a wall thickness of 0.65 to 0.7 mm as a mean value measured four times around the circumference of the tube, and a gage length of 30 to 50 mm. The volume fraction of fibers was about 58%. Average maximum strength and initial modulus was found to be 724 MPa and 34 GPa for tension, and 88 MPa and 2.8 GPa for torsion, respectively. The biaxiality ratios, which is defined as the ratio of axial stress, σ, to shear stress, τ, used here were 2/1, 1/1 and 0/1 (pure torsion). The biaxial stress state can be attained when the tubular specimen is simultaneously twisted along its longitudinal axis while it is subjected to axial loading. Pure torsion tests were performed with a rate of 0.12 degree/min under angle control, where axial force was controlled to be zero. In biaxial tests, load and torque control mode were adopted to control biaxiality ratios. The loading rates were 100 to 500 N/s for tension. The loading rate for torsional loading was a value depending on the biaxiality ratio.

To compare the equations (2) and (5) for CFRP, the strains were measured from three strain gage rosettes attached in the center of the gage length section. Elastic modulus of CFRP was calculated using the strain gage data while the large strains were measured using the output of LVDT and ADT of the MTS with equation (6).

4. RESULTS AND DISCUSSION

Pure torsion tests were performed using a CFRP tube having three strain gage rosettes attached in the center of the gage length section. Figure 3 shows the comparison of shear strains obtained by equations (2c) and (5c) respectively, and calculated shear strain using equation (6b). In Fig. 3, an extrapolation for shear strains by strain gage was conducted up to failure strain of specimen using neural networks [12]. The strain

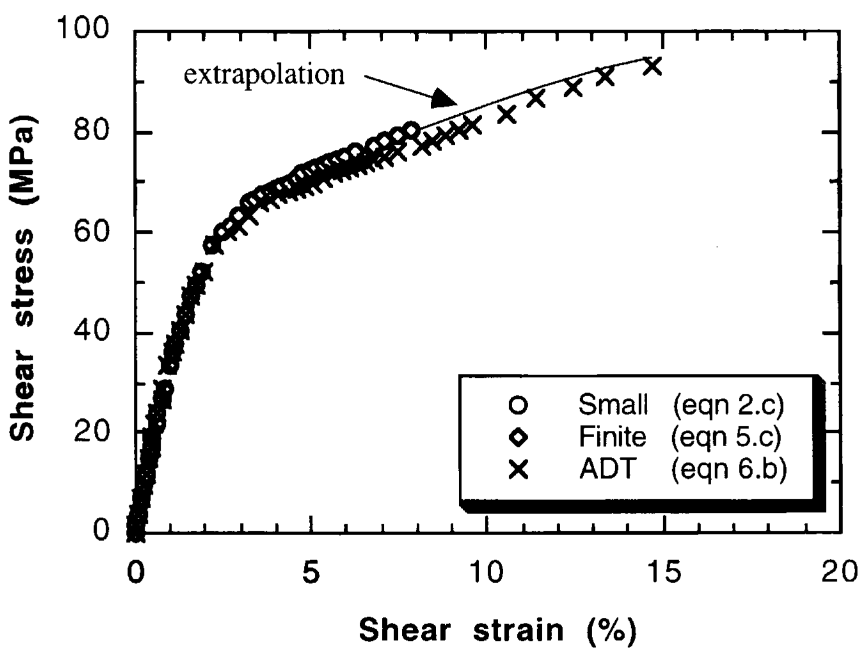

Figure 3. Comparison of shear strain under pure torsional loading.

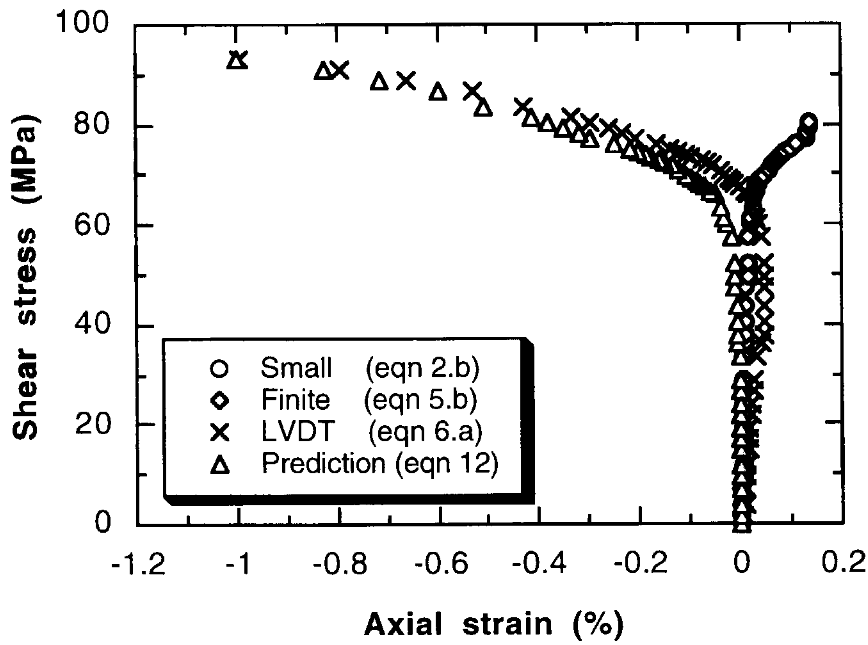

Figure 4. Comparison of axial contraction with prediction under pure torsional loading.

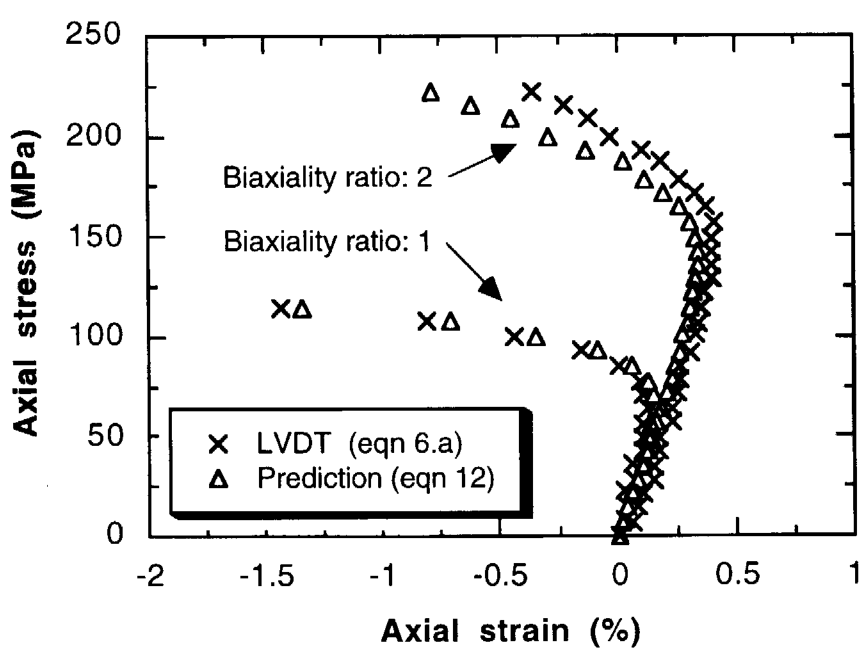

Figure 5. Comparison of axial contraction with prediction under biaxial loading.

measurement using strain gage was terminated at the shear strain of 8% due to the debonding of strain gages in the experiment. As can be seen in Fig. 3, the calculated strains using the strain gage measurements and equations (2c) and (5c) agree well with the ADT measurement using equation (6b). It means that there is no slip between the hydraulic grip and specimen. Another interesting aspect is the fact that the strain calculated by equation (2c) derived for infinitesimal deformation also provides a good result. Actually the difference between the strains computed by using equations (2c) and (5c) respectively are almost indistinguishable up to 8% shear strain because the effect of second order terms introduced in equation (5c) is almost negligible. The equation derived in this study, however, must be used when strains are large.

It is noteworthy fact that axial contraction of a tube occurred at lower biaxiality ratios of tension/torsion biaxial loading, though the specimen was under tensile loading as well as torsion. In Fig. 4, the axial strain using strain gage measurement with equation (2b) is almost same as the strain with equation (5b) at the early stage of loading. However, it could not present the axial contraction of a tube measured by the LVDT of testing machine, as can be seen in Fig. 4. That is, while the strain gage mounted in 90 degrees registered only extension, actually contraction of a tube in the axial direction occurred. The prediction of axial contraction of a tube using equation (12) is in good agreement with the result of LVDT of the testing machine. The prediction of axial contraction of a tube for biaxiality ratio of 1 and 2 is compared with experimental data in Fig. 5. The comparison shows that equation (12) could predict axial contraction of a tube much better for a biaxiality ratio of 1 than for a biaxiality ratio of 2. From Figs 4 and 5, it can be said that the equation (12) gives the better prediction for smaller biaxiality ratio under tension/torsion biaxial loading. Axial contraction of a cross-ply CFRP tube by torsional loading may occur because of the presence of longitudinal fiber in FRP, while such contraction did not occur in general isotropic materials because of its shearing phenomenon by grain boundary slip. From the comparison of equation (12) with the result using Hooke's law, where shear modulus G is considered to be constant, instead of the twist angle, γ' in equation (12), it was found that the turnaround of the shear stress/shear strain curve to the opposite direction is originated due to larger degradation of torsional rigidity of tube than that of axial rigidity as the biaxial loads increase. As a result, axial contraction of a tube can appear.

5. CONCLUSIONS

The general equation for calculating strains from strain gage measurements for the case of large deformation and large rotation was derived based on the analysis of kinematics of deformation. This equation can also represent the well-known equation established for infinitesimal deformation. The signal of LVDT and ADT could be adopted in strain measurement of the biaxial test for large deformation, if a no slip condition can be provided between the hydraulic grip and specimen.

Axial contraction of a cross-ply tube may be originated due to the presence of longitudinal carbon fiber in CFRP and the degradation of torsional rigidity of a tube as the biaxial loads increase, which could be predicted as a function of stresses, a twist angle and an elastic modulus. The prediction of axial contraction of a CFRP tube is in good agreement with the result of LVDT of testing machine under tension/torsion biaxial loading.

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APPENDIX: GOVERNING EQUATION FOR THE MEASUREMENT OF LARGE STRAINS

Consider a two dimensional rectangular block OABC with unit length in each direction. Let the block be subjected to shear deformations as well as stretches in both the X_1 and X_2 directions, as shown in Fig. 6. The general description of deformation

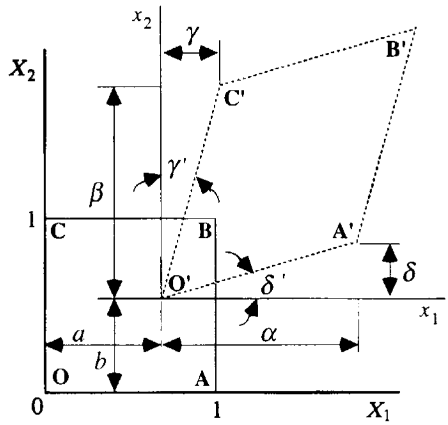

Figure 6. General deformation of unit block under multi-axial loading.

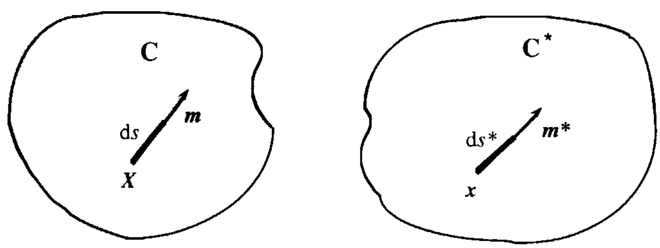

Figure 7. Reference configuration **C** and deformed configuration **C***.

mode is

$$x_1 = a + \alpha X_1 + \gamma X_2, \tag{A1}$$

$$x_2 = b + \delta X_1 + \beta X_2, \tag{A2}$$

where X_1 and X_2 are the reference (undeformed) coordinates of any point in the block, and x_1 and x_2 are the current (deformed) coordinates of the same point. The a and b are a parallel motion, and α and β are the stretch of the unit block in the X_1 and X_2 direction, respectively. δ and γ are the shear deformations as given linear displacement, which is uniquely related to the usual twist angle δ' and γ' through the relation, $\tan\delta' = \delta/\alpha$ and $\tan\gamma' = \gamma/\beta$, respectively. In the case of infinitesimal deformation and pure shear ($\alpha = 1$ and $\gamma = 0$, or $\beta = 1$ and $\delta = 0$), δ and γ converge to the usual infinitesimal shear strains.

Now, consider an arbitrarily oriented infinitesimal fiber element $\mathrm{d}\boldsymbol{X}$ in the undeformed configuration, which becomes $\mathrm{d}\boldsymbol{x}$ in the deformed configuration [2, 7]. Let $\boldsymbol{m}$ be a unit vector and a fiber is $\mathrm{d}s\boldsymbol{m}$ in the undeformed configuration, as shown in Fig. 7. Suppose that its image in the deformed configuration is $\mathrm{d}s^*\boldsymbol{m}^*$. Thus a directional vector of the fibers can be given as follows:

$$\mathrm{d}\boldsymbol{X} = \mathrm{d}s\boldsymbol{m}, \tag{A3}$$

$$\mathrm{d}\boldsymbol{x} = \mathrm{d}s^*\boldsymbol{m}^*. \tag{A4}$$

The two configurations are related by

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{F}\mathrm{d}\boldsymbol{X}, \tag{A5}$$

where $\boldsymbol{F}$ is deformation gradient tensor, which can be calculated from equations (A1) and (A2) to be

$$[\boldsymbol{F}_{ij}] = \left[\frac{\partial x_i}{\partial X_j}\right] = \begin{bmatrix} \alpha & \gamma \\ \delta & \beta \end{bmatrix} \quad i, j = 1, 2. \tag{A6}$$

The nominal strain in the $\boldsymbol{m}$ direction is defined by

$$\varepsilon_\theta = \frac{\mathrm{d}s^* - \mathrm{d}s}{\mathrm{d}s}. \tag{A7}$$

This is the strain which can be measured directly by a strain gage mounted along the $\boldsymbol{m}$ direction in the undeformed configuration. Equations (A3), (A4), (A5) and (A7) give

$$(1 + \varepsilon_\theta)^2 = \boldsymbol{m}^T \cdot \left[\left(\boldsymbol{F}^T \cdot \boldsymbol{F}\right)\right] \cdot \boldsymbol{m}, \tag{A8}$$

where,

$$\boldsymbol{F}^T \cdot \boldsymbol{F} = \begin{bmatrix} \alpha^2 + \delta^2 & \alpha\gamma + \beta\delta \\ \alpha\gamma + \beta\delta & \beta^2 + \gamma^2 \end{bmatrix}.$$

Let $\boldsymbol{m} = [\cos\theta \sin\theta]^T$; and then substitution of $\boldsymbol{m}$ and $\boldsymbol{m}^T$ into equation (A8) gives

$$(1 + \varepsilon_\theta)^2 = \left(\alpha^2 + \delta^2\right)\cos^2\theta + \left(\beta^2 + \gamma^2\right)\sin^2\theta + 2(\alpha\gamma + \beta\delta)\sin\theta\cos\theta, \tag{A9}$$

where θ is the inclined angle of the strain gages to the x-axis in the undeformed configuration and ε_θ is the strain measured by strain gage inclined at θ degree to the x-axis in the undeformed configuration. α, β, γ and δ are the elements of the deformation gradient tensor, which can be given as a function of measured strains. Incidentally, two equations among the four equations to find four unknowns in equation (A9) are dependent on each other because a component of rotation is included in γ and δ. Thus letting $\delta = 0$ in equation (A9) to eliminate the component of rotation, which is satisfied directly in the case of tube deformation, equation (A9) becomes

$$(1 + \varepsilon_0)^2 = \alpha^2\cos^2\theta + \left(\beta^2 + \gamma^2\right)\sin^2\theta + 2\alpha\gamma\sin\theta\cos\theta. \tag{A10}$$

It is clear that three independent measurements have to be made in order to completely determine a deformation mode because of three unknowns α, β and γ. What has to be noticed is that equation (A10) is valid for large deformation and large rotation.

For infinitesimal deformation, substitution of $\alpha = 1 + e_x$, $\beta = 1 + e_y$, and $\gamma = 2e_{xy}$ into equation (A10) and omission of second order terms give the well-known equation;

$$e_\theta = e_x \cos^2\theta + e_y \sin^2\theta + 2e_{xy} \sin\theta \cos\theta, \tag{A11}$$

where e_x, e_y and e_{xy} are the Cartesian components of the infinitesimal strain tensor, which can be given by three independent measurements of nominal strains measured by strain gage inclined at different degree in the undeformed configuration.